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ANALYSIS OF GEOMETRICALLY NONLINEAR PLANE FRAMES INCORPORATING SHEAR DEFORMATIONS

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Abstract—The state of equilibrium of plane bars and frames is formulated with finite deflections and shear deformations taken into consideration. The derivation is based on continuum solid mechanics, with integration applied to the original undeformed length.

This paper presents a brief resume of a geometrically nonlinear analysis of plane slender bars and frames[1, 2] founded on continuum solid mechanics derivation, incorporating moderate shear deformations and allowing for finite displacements. Expressions for deformations are exact within known engineering assumptions, valid in case of slender bars.

The bar is regarded as one dimensional continuum, initially straight in the reference state. Each point along its deformed axis is characterized by u_0 and w_0 (displacement components of the geometrical axis displacement vector $\mathbf{U} = \mathbf{R} - \mathbf{r}$ in terms of the position vectors in the deformed and undeformed states respectively), also by $\bar{K}$ $(= \mathrm{d}\Psi/\mathrm{d}s)$ curvature, in terms of the section rotation.

The state of equilibrium of the deformed bar, under the action of end forces H, V, M and distributed conservative load $\mathbf{f} = f_x\mathbf{i} + f_z\mathbf{k}$, is governed by the variational equation (Fig. 1):

$$-\iiint_{\bar{v}} \sigma^{ij}\delta\eta_{ij}\,\mathrm{d}\bar{v} - H_A\delta u_{0_A} + H_B\delta u_{0_B} - V_A\delta w_{0_A} + V_B\delta w_{0_B} + M_A\delta\Psi_A - M_B\delta\Psi_B + \int_L \mathbf{f}\cdot\delta u\,\mathrm{d}_L = 0. \quad (1)$$

Equation (1) is transformed to the undeformed volume employing the principle of mass conservation, and the components of the stress tensor σ^{ij} resolved to components with physical dimensions $\sigma^{(ij)}$[2, 3].

The Lagrangian strain tensor η_{ij}, given in terms of the covariant components of the metric tensors G_{ij} and g_{ij} (in the deformed and undeformed states respectively) is actually in terms of the displacement derivatives being derived from the position vector R in the deformed state:

$$\mathbf{R} = [(x + u_0) - z\sin\Psi]\mathbf{i} + [w_0 + z\cos\Psi]\mathbf{k}. \quad (2)$$

Thus, equation (1) yields three nonlinear differential equations of equilibrium and six boundary conditions for the three unknown displacements. The equations are transformed to polynomial form through a suitable mathematical substitution[2, 3].

Establishment of force equilibrium and displacement compatibility at a joint of two bars permits the consideration of several bars to form a frame analysis[2, 3].

Numerical solution is obtained by transforming the nonlinear polynomial differential

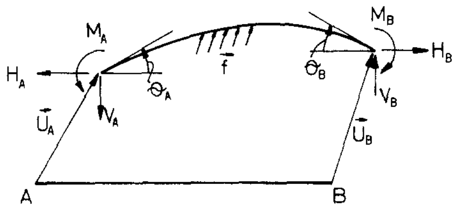

Fig. 1.

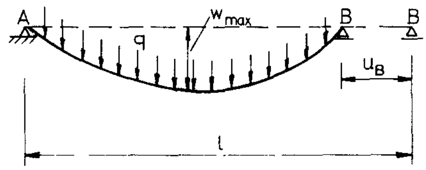

Fig. 2.

Table 1

$\alpha = \frac{ql^3}{EI}$	K_{max}	$\left[\frac{w}{l}\right]_{max}$	Increase in deflection due to shear deformations	$\left[\frac{u}{l}\right]_B$	$\beta = \frac{1}{\alpha}\left[\frac{w}{l}\right]_{max}$
0·5	0·05207 × 10^{-2}	0·00651	2·50%	0·000103	0·013018
	(0·05207)	(0·00667)		(0·000108)	(0·013344)
1·0	0·10411	0·01301	2·50%	0·000410	0·013011
	(0·10411)	(0·01334)		(0·000432)	(0·013336)
2·0	0·20786	0·02596	2·48%	0·001634	0·012980
	(0·20784)	(0·02660)		(0·001722)	(0·013302)
3·0	0·31091	0·03878	2·46%	0·003652	0·012928
	(0·31082)	(0·03974)		(0·003847)	(0·013246)
4·0	0·41291	0·05143	2·43%	0·006431	0·012857
	(0·41271)	(0·05268)		(0·006771)	(0·013169)
5·0	0·51358	0·06384	2·39%	0·009930	0·012768
	(0·51319)	(0·06536)		(0·010447)	(0·013073)
6·0	0·61265	0·07597	2·34%	0·014096	0·012662
	(0·61199)	(0·07775)		(0·014819)	(0·012958)
7·0	0·70986	0·08779		0·018872	0·012541
8·0	0·80504	0·09926		0·024197	0·012407
9·0	0·89804	0·11035		0·030007	0·012261
10·0	0·98872	0·12105		0·036239	0·012105
11·0	1·07700	0·13136		0·042830	0·011941
12·0	1·16284	0·14125		0·049721	0·011771
13·0	1·24621	0·15074		0·056856	0·011595
14·0	1·32712	0·15983		0·064185	0·011416
15·0	1·40565	0·16852		0·071668	0·011235
16·0	1·48163	0·17682		0·079242	0·011051
17·0	1·55539	0·18476		0·086897	0·010868
18·0	1·62678	0·19233		0·094580	0·010685
19·0	1·69594	0·19955		0·102268	0·010503
20·0	1·76294	0·20644		0·109939	0·010322

†Figures in parentheses represent computed values with shear deformations taken into consideration.

equations to simultaneous nonlinear algebraic equations. Newton Raphson's iterative procedure for solution of a set of nonlinear algebraic equations is employed proving to be a powerful tool for solution of a large number of unknowns.

The contribution of shear deformations is small in bars and even smaller in frames. This contribution diminishes with increased nonlinearity. Convergence of the mathematical procedure is obtained for very large displacements (up to about 40 per cent of the span in several occasions) and for relative smaller values, however, when shear deformations are considered.

NUMERICAL EXAMPLE

Simply supported, uniformly loaded beam *AB* (Fig. 2) (181 points grid) is loaded by uniformly distributed load q. Maximum deflection at mid point is $w_{max} = \beta\alpha$ where $\alpha = ql^3/EI$. w_{max}, K_{max} and u_B—horizontal movement at support B, are listed in Table 1. Contribution of shear deformations is indicated in parallel (Table 1).

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